WITHDRAWAL RATES, BUFFER PORTFOLIOS, AND ASSET ALLOCATION: SIMULATION RESULTS

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ABSTRACT

The appropriate size of the sustainable withdrawal rates from portfolios of assets intended to fund investors' retirement years has attracted the attention of several researchers over the years (Bengen, 1994, 1996, 1997; Cooley, Hubbard, and Walz [hereafter CHW], 1998, 1999). Recently, Ogborne and Woerheide [hereafter OW] (2001) introduced the concept of a "buffer portfolio" into the study of withdrawal rates. This paper examines OW's work, but is unable to reproduce their results demonstrating superior performance when buffer portfolios are used. This paper then uses Monte Carlo simulation to generate statistically robust descriptions of the optimum asset allocation to sustain various withdrawal rates over periods of twenty, twenty-five, and thirty years.

We find that for low withdrawal rates (5% per year or less), success rates are relatively high without regard to the proportion of retirement assets invested in equities versus debt. On the other hand, for high withdrawal rates (11% or 12%), the optimum asset allocation is 100% equity, but success rates generally fall below 50% even at the optimum 100% equity allocation. For withdrawal rates of 6% and 7%, the evidence is ambiguous. However, for withdrawal rates between 8% and 10% inclusive, the optimum asset allocation approaches 100% equities and there is a high correlation between success rates and the proportion of the portfolio invested in equities. We conclude that asset allocation is unimportant at low withdrawal rates and pointless at high withdrawal rates (unless clients are willing to accept success rates below 50%). For withdrawal rates from 8% to 10%, asset allocation is the crucial determinant of success and the optimum allocation is at or near to 100% equity.

INTRODUCTION

William Bengen wrote a series of three articles (1994, 1996, 1997) for the *Journal of Financial Planning* in which he discussed asset allocation for retirement portfolios. Bengen's analysis led him to suggest that retirees should place a relatively large proportion of their assets in equities. He suggested perhaps 75% in the early years of retirement, declining as one ages according to rules such as "% of portfolio in stocks = 133 minus your age." CHW (1998, 1999) developed a procedure for measuring the tradeoffs between withdrawal rates from the portfolio, expected years of retirement, and asset allocation. Using historical data provided by Ibbotson and Associates, CHW generate an algorithm that grows an initial portfolio amount by the total return appropriate to a given asset allocation and reduces the portfolio value by a fixed withdrawal amount. For example, one might start with a retirement portfolio of $100,000, a withdrawal rate of 7% (i.e., $7,000 per year), and an asset allocation of 50% stocks and 50% long-term high-grade corporate bonds. Suppose the total return on stocks is 8% in the first year of retirement and the total return on bonds is 4%. A 50/50 portfolio would then grow at 6% to a value of $106,000. The retirement income withdrawal of $7,000 would then be taken from the retirement fund and the remaining $99,000 invested for the next period. This continues until the desired length of retirement is met. Success is then defined as having a non-negative value for the portfolio at the end of the retirement period (i.e., not outliving your money). CHW's 1998 paper used annual withdrawals and, implicitly, annual rebalancing. CHW's 1999 paper used monthly withdrawals and implicit rebalancing.

Building on CHW, OW (2001) demonstrate that by placing a portion of one's retirement portfolio into money market funds as a "buffer portfolio," one can sustain a given withdrawal rate with greater likelihood of success. Success is again defined as not running out of money by the end of a defined period of time, say, twenty years. In the OW model, placing one year's worth of withdrawals into a buffer portfolio achieved improved performance. The idea is that if the market goes down, withdraw the money from the buffer; else, withdraw the year's retirement money from the investment portfolio. OW explain their results as being consistent with modern portfolio theory and classic risk-return tradeoffs. They state:

> When cash is being steadily withdrawn from a portfolio, then the reduction in the variability of the returns [accomplished by placing a portion of the assets into a money market fund] … reduces the chances of a series of poor returns … depleting the portfolio. This is good. However, the associated reduction in the expected returns reduces the ability of the portfolio to grow in value, and thus keeps the portfolio at risk of being subsequently depleted. This is bad.

However, our results do not support OW. We find no benefit to using such buffer portfolios. We then extend the CHW and OW studies by using Monte Carlo simulation to provide robust estimates of the relationship between success rates and asset allocation.

In the next section, we discuss the data and methodology used in our study. We then present results of our replications of the CHW and OW studies. We follow with our simulation results and end the paper with a short summary and concluding remarks.

DATA AND METHODOLOGY

Our methodology is based on that of CHW and OW. We start with an initial endowment of $100,000 and grow that endowment by a periodic total return factor that depends on one of five asset allocations tested. As in both CHW and OW, the five asset allocations tested are 100% equity, 75% equity and 25% bonds, 50% of each, 25% equity and 75% bonds, and 100% bonds.

At the end of each period we withdraw a fixed amount from the portfolio. We test ten withdrawal rates from 3% ($3,000 per year) to 12% ($12,000 per year). For monthly tests, the withdrawal rates vary from $250 per month (3% yearly rate) to $1,000 per month (12% yearly rate). We define success to occur if at the end of a given time period (we use 20, 25, and 30 year periods), the portfolio has a non-negative value.

The total return factors use historical data on common stock total returns, corporate bond total returns, long-term government bond total returns, and Treasury bill total returns taken from the Ibbotson and

Associates Yearbook (2000). As reported below, we are able to closely replicate the two CHW papers. CHW's first paper used annual returns, annual withdrawals, and, implicitly, annual rebalancing of the portfolio and covered the time period from 1926 to 1995. Their second paper used monthly returns, withdrawals, and rebalancing and covered the time period from 1926 through 1927. Both CHW papers used corporate bonds as their debt instruments.

We also attempt to replicate the OW results. OW used annual returns, annual withdrawals, and annual rebalancing and covered the period from 1926 to 1998. They use long-term government bonds as their debt vehicle. OW also use a "buffer portfolio" in their study. They place from one to four years' worth of withdrawals into a buffer portfolio conceptually made up of a money market mutual fund. They proxy the MMMF return by using T-bill total returns. Under OW's approach, when the return on the investment portfolio is positive, the withdrawal is taken from the investment portfolio and all interest from the buffer portfolio is transferred to the investment portfolio. However, should the return on the investment portfolio be negative, the withdrawal is taken from the buffer portfolio. If the buffer portfolio has been depleted, the withdrawal is taken from the investment portfolio. When the investment portfolio shows a positive return following at least one negative return, the withdrawal is taken from the investment portfolio and the buffer portfolio is replenished. OW test buffer portfolios of from one to four years of withdrawals. However, in this paper, we look only at buffer portfolios of one year's withdrawals. Our replications are based on the same time periods as the respective CHW or OW studies. We use corporate bonds to replicate the CHW works and long-term government bonds to replicate OW's efforts.

Our simulations are based on the time period from 1926 through 1999 inclusive. Our simulations use long-term government bonds. For each year and withdrawal rate combination we perform, independently, 101 Monte Carlo simulations of 10,000 replications. These 101 simulations cover asset allocations from 100% equity to 0% equity in increments of one percentage point. Thus, we perform 3,030 simulations of 10,000 replications each, a total of 30,300,000 replications. Each replication randomly selects, with replacement, the correct number of months for the length of time in that simulation from the 888 months in the time period of the study. For example, there are 360 months in a thirty year period, so that one replication of a thirty year analysis for a given withdrawal rate and asset allocation consists of choosing 360 months, one month at a time, with replacement, from the 888 months in the data. For asset allocations that mix stocks and bonds, a paired set of returns is randomly selected at each selection (i.e., if May, 1951 is randomly selected, then both the stock return and the bond return for that simulated month will be those of May, 1951).

To examine the relationship between success and asset allocation, we run nonparametric correlations (Kendall's Tau and Spearman's Rho) of the percentage of successes per 10,000 replications against the asset allocations. We run these correlations tests separately for each year and withdrawal rate combination. Nonparametric techniques are indicated as the distributions of these success rates are far from normal (results of these tests for normality are available from the authors).

REPLICATION RESULTS

Table 1 presents a comparison of CHW, OW, and this current paper. Table 1 is based on historical data in actual order of occurrence (i.e., not random selection). Column 1 gives the years in the retirement period, column 2 is the withdrawal rate, and columns 3 and 4 are our results of replicating the CHW and OW studies for the 100% equity allocation. Column 5 reports the difference, Ä, between our results and those of OW. For example, For 20 years, 4% withdrawal rate, and 100% equity allocation we calculate a 98% success rate. For the same combination, OW report a success rate of 100%. Our result is two percentage points below that of OW; hence we report a –2 in column five for 20 years, 4% withdrawal rate, and 100% equity allocation. Columns 6 through 8 report the results for the 75% equity, 25% debt allocation, columns 9 through 11 report the 50% equity results, columns 12 through 14 the 25% equity results, and columns 15 through 17 report the 100% debt allocation results

We do not report the differences between our replication and the results reported by CHW as only one such difference exists. At 20 years, 10% withdrawal rate, and 100% equity allocation, we report a success rate of 59% while CHW report a success rate of 61%. All remaining success rates in our replication exactly match those reported by CHW. We are comfortable in stating that we succeeded in replicating CHW's 1998 (annual) results.

Such is not the case in regards to OW, however. There is substantial variance between our results using the buffer portfolio approach and OW's corresponding results. Our success rates are often lower. At the higher withdrawal rates, these differences are substantial, with the maximum difference in Table 1 being -34 at 20 years, 8% withdrawal rate, and a 25% equity allocation. For this combination, we report a success rate of 46%, but OW report a success rate of 80%. We clearly have not replicated OW, but are at a loss as to how to explain the divergence in the two sets of results. This divergence explains the diametrically opposite conclusions we draw regarding the utility of using a buffer portfolio, as compared to the conclusions reached by OW. OW report that across all combinations of allocations, years, and withdrawal rates that they test, the buffer approach is more successful than the no buffer approach by a score of 63 to 4 (out of the 140 combinations they test). However, as a close reading of Table 1 reveals, we find the score to be 8 to 74 (out of 150 total combinations).

Table 2 presents results based on actual calendar order monthly analysis across the period from 1926 through 1999. The columns under the symbol Ö in Table 2 reports the differences between the no buffer and with buffer success rates. Some of the differences are dramatic, with the greatest difference, -15 percentage points, occurring at the 25 years, 9% withdrawal rate, and 50% equity allocation combination. The no buffer approach was more successful than the with buffer approach by a score of 78 to 2. In stark contrast to the results reported by OW, our replication results clearly indicate that placing one year's worth of withdrawals into a buffer portfolio and withdrawing the needed funds from the buffer portfolio if the market declines is not a superior strategy.

SIMULATION RESULTS

Table 3 reports the minimum and maximum success rates derived from our simulations and their associated asset allocations. For example, for 20 years of withdrawals at a 3% withdrawal rate, the minimum success rate from the simulation was 99.73%, which occurred at an asset allocation of 0.99 (i.e., 99% equity, 1% long - term government bonds). At that same year rate combination, the maximum

Table 1
Comparison of This Paper to CHW and OW
Annual Withdrawals – Actual Calendar Order
1926 through 1995 (CHW)
1926 through 1998 (OW)

		100% Equity			75% Equity			50% Equity			25% Equity			0% Equity		
Years	Rate	No Buffer (CHW)	Buffer (OW)	Δ	No Buffer (CHW)	Buffer (OW)	Δ	No Buffer (CHW)	Buffer (OW)	Δ	No Buffer (CHW)	Buffer (OW)	Δ	No Buffer (CHW)	Buffer (OW)	Δ
20	3	100	100		100	100		100	100		100	100		100	100	
25	3	100	100		100	100		100	100		100	100		100	100	
30	3	100	100		100	100		100	100		100	100		100	100	
20	4	98	98	-2	100	100		100	100		100	100		100	100	
25	4	98	98	-2	100	100		100	100		100	100		100	100	
30	4	98	98		100	100		100	100		100	100		100	98	-2
20	5	96	96		100	98	-2	100	100		100	100		100	100	
25	5	96	96		98	98	-2	100	100		100	100		98	86	-10
30	5	95	93	-3	98	95	-5	100	100		100	100		51	52	-9
20	6	94	94		96	96	-2	100	98	-2	100	100		96	80	-16
25	6	91	94		96	94	-2	98	96	-4	100	100		52	39	-14
30	6	90	91	-2	95	93	-3	98	93	-7	100	84	-16	27	5	-31
20	7	92	91	-3	94	94		96	94	-4	100	96	-4	47	41	-16
25	7	87	86	-4	91	90	-4	96	90	-6	78	53	-29	26	14	-25
30	7	85	84	-5	88	86	-7	90	82	-11	32	9	-27	0	2	-21
20	8	84	85	-6	88	87	-7	88	81	-13	71	46	-34	35	22	-22
25	8	78	78	-6	78	76	-8	70	55	-21	22	16	-13	7	8	-19
30	8	78	75	-5	73	68	-7	51	43	-9	5	0	-18	0	0	-2
20	9	73	72	-10	71	67	-11	61	56	-11	24	22	-11	16	11	-21
25	9	70	61	-13	57	53	-8	43	35	-14	9	6	-14	2	0	-8
30	9	68	59	-7	54	48	-9	37	18	-16	0	0	-11	0	0	-2
20	10	59	61		51	48		41	37		12	11		6	2	
25	10	50	47		46	35		22	10		0	0		0	0	
30	10	54	45		46	34		15	0		0	0		0	0	
20	11	47	46		41	41		25	13		4	6		0	0	
25	11	43	37		33	27		7	0		0	0		0	0	
30	11	49	39		37	23		0	0		0	0		0	0	
20	12	43	43		33	31		10	7		2	2		0	0	
25	12	35	29		26	14		0	0		0	0		0	0	
30	12	34	32		24	9		0	0		0	0		0	0	

Table 2
Comparison of No Buffer to Buffer Approaches
Monthly Withdrawals – Actual Calendar Order
1926 through 1999

		100% Equity			75% Equity			50% Equity			25% Equity			0% Equity		
Years	Rate	No Buffer	Buffer	Φ	No Buffer	Buffer	Φ	No Buffer	Buffer	Φ	No Buffer	Buffer	Φ	No Buffer	Buffer	Φ
20	3	100	100		100	100		100	100		100	100		100	100	
25	3	100	100		100	100		100	100		100	100		100	100	
30	3	100	100		100	100		100	100		100	100		100	100	
20	4	98	98		100	100		100	100		100	100		100	100	
25	4	97	97		100	100		100	100		100	100		100	100	
30	4	97	97		100	100		100	100		100	100		100	100	
20	5	96	96		99	98	-1	100	100		100	100		100	100	
25	5	95	95		97	97		100	100		100	100		81	83	+2
30	5	94	93	-1	97	96	-1	100	99	-1	100	100		51	49	-2
20	6	94	93	-1	96	96		100	99	-1	100	100		79	80	+1
25	6	92	92		95	94	-1	97	96	-1	100	99	-1	36	34	-2
30	6	91	90	-1	93	92	-1	95	94	-1	85	79	-6	8	8	
20	7	91	90	-1	94	92	-2	96	94	-2	93	92	-1	41	38	-3
25	7	88	87	-1	91	89	-2	88	85	-3	46	35	-11	14	14	
30	7	85	84	-1	88	85	-3	74	69	-5	11	8	-3	3	3	
20	8	84	83	-1	85	83	-2	78	75	-3	47	33	-14	21	20	-1
25	8	77	75	-2	72	69	-3	56	48	-8	15	13	-2	8	8	
30	8	72	69	-3	63	57	-6	41	32	-9	2	1	-1	1	1	
20	9	72	69	-3	68	62	-6	53	46	-7	18	17	-1	13	13	
25	9	62	57	-5	52	44	-8	33	18	**-15**	4	4		2	2	
30	9	59	51	-8	46	39	-7	13	5	-8	0	0		0	0	
20	10	60	54	-6	51	45	-6	36	22	-14	11	11		4	4	
25	10	48	42	-6	38	32	-6	9	3	-6	2	2		0	0	
30	10	47	40	-7	36	27	-9	2	1	-1	0	0		0	0	
20	11	49	43	-6	41	35	-6	16	10	-6	5	4	-1	0	0	
25	11	39	32	-7	29	17	-12	2	1	-1	0	0		0	0	
30	11	39	33	-6	24	10	-14	0	0		0	0		0	0	
20	12	42	35	-7	33	19	-14	6	5	-1	1	0	-1	0	0	
25	12	32	28	-4	17	7	-10	1	0	-1	0	0		0	0	
30	12	33	26	-7	10	3	-7	0	0		0	0		0	0	
Greatest difference				-8			-14			-15			-14			-3

Table 3
Equity Allocation and Success
Monte Carlo Simulation

Years	Rate	Minimum	Allocation	Maximum	Allocation
20	3%	99.73	99	100.00	72
25	3%	99.62	97	100.00	59
30	3%	99.37	100	100.00	61
20	4%	99.21	96	100.00	58
25	4%	98.62	100	100.00	35
30	4%	98.08	100	100.00	32
20	5%	97.48	100	99.95	29
25	5%	95.73	0	99.56	26
30	5%	89.12	0	98.76	29
20	6%	93.78	0	99.11	32
25	6%	78.99	0	96.39	47
30	6%	61.04	0	93.92	55
20	7%	74.58	0	94.68	39
25	7%	48.90	0	88.89	70
30	7%	30.74	0	85.38	77
20	8%	45.16	0	85.85	56
25	8%	21.94	0	79.58	95
30	8%	12.20	0	76.21	97
20	9%	21.05	0	76.49	84
25	9%	8.07	0	71.64	100
30	9%	3.59	0	67.41	99
20	10%	7.74	0	66.99	96
25	10%	2.43	1	61.35	99
30	10%	0.77	0	57.65	98
20	11%	2.38	0	58.03	98
25	11%	0.57	1	51.51	100
30	11%	0.10	0	47.39	97
20	12%	0.65	7	48.8	100
25	12%	0.11	0	43.03	100
30	12%	0.03	3	39.39	99

success rate is 100%, which occurred at an asset allocation of 72% equity and 28% long-term government bonds. Note that for withdrawal rates of 5% or less, the difference between the minimum and maximum success rates is relatively small. Thus, if you choose such small withdrawal rates, there is little chance you'll "outlive your money" regardless of the asset allocation you adopt.

However, for higher withdrawal rates, there is substantial variation from the minimum success rate to the maximum success rate. Withdrawal rates of 9% and higher show maximum success at allocations that equal or approach 100% equity. Also of interest is the fact that for withdrawal rates of 11% and 12%, maximum success rates are near 50% or even lower. If you choose such high withdrawal rates, there is a better than even chance that you will run out of money before you run out of time.

The 6%, 7% and 8% withdrawal rates results are more of a mixed bag. Clearly, minimum success is associated with a zero equity allocation, but the maximum success rates are found at equity allocations ranging from 32% to 97%. Trends are apparent, though. As one moves to a longer time frame within a given withdrawal rate, a greater equity allocation is indicated. And as one moves to higher withdrawal rates, achieving maximum success also requires one to increase the equity allocation in the investment portfolio

Table 4 confirms these trends and provides statistical tests of the strength of the relationship between success and equity allocation. The correlations are negative and significantly different from zero for withdrawal rates of 3%, 4%, and for the 20 and 25 year period at the 5% withdrawal rate. Then there is sort of a "transition zone" at the 6% and 7% withdrawal rates. The correlations move from negative and significant to near zero (and not significantly different from zero) to positive and significant. The correlations for all withdrawal rates above 7% are positive and significantly different from zero. Furthermore, at the 9% withdrawal rate and higher, the correlations are, essentially, perfect positive correlations. Confirming the evidence presented in Table 3, the mean and median success rates for withdrawal rates of 10% and higher are near or below 50%. The standard deviations of the success rates move from near zero at low withdrawal rates to values exceeding one-half the mean success rate at withdrawal rates of 10% and higher.

SUMMARY AND CONLCUDING REMARKS

We find that for low withdrawal rates (5% per year or less), success rates are relatively high without regard to the

Table 4
Correlation between Success and Equity Allocation
Monte Carlo Simulation

Year	Rate	Mean	Median	Std. Dev.	Spearman	Kendal
20	3%	99.98	100.00	0.05	-0.79 *	-0.66 *
25	3%	99.95	100.00	0.10	-0.88 *	-0.76 *
30	3%	99.92	99.99	0.14	-0.85 *	-0.73 *
20	4%	99.87	99.99	0.21	-0.88 *	-0.78 *
25	4%	99.70	99.87	0.37	-0.82 *	-0.69 *
30	4%	99.44	99.65	0.52	-0.67 *	-0.56 *
20	5%	99.37	99.66	0.67	-0.81 *	-0.68 *
25	5%	98.39	98.79	1.03	-0.49 *	-0.45 *
30	5%	96.89	97.49	1.98	-0.14	-0.21 *
20	6%	97.38	97.75	1.39	-0.47 *	-0.44 *
25	6%	93.35	94.61	3.71	0.10	- 0.01
30	6%	88.87	91.80	7.38	0.44 *	0.27 *
20	7%	91.09	92.49	4.20	0.13	0.01
25	7%	81.90	86.94	10.26	0.67 *	0.52 *
30	7%	74.44	82.77	15.12	0.88 *	0.76 *
20	8%	78.33	83.89	10.67	0.75 *	0.59 *
25	8%	65.41	74.28	17.29	0.98 *	0.91 *
30	8%	57.46	68.84	20.56	1.00 *	0.97 *
20	9%	61.98	70.48	16.85	0.97 *	0.90 *
25	9%	49.06	57.31	20.76	1.00 *	0.97 *
30	9%	42.20	49.13	21.90	1.00 *	0.98 *
20	10%	45.99	54.01	19.85	1.00 *	0.97 *
25	10%	35.42	40.04	20.68	1.00 *	0.99 *
30	10%	30.27	33.60	20.31	1.00 *	0.99 *
20	11%	32.83	36.64	19.44	1.00 *	0.99 *
25	11%	24.94	25.49	18.25	1.00 *	0.99 *
30	11%	21.15	20.24	17.15	1.00 *	0.99 *
20	12%	23.11	22.72	17.10	1.00 *	0.99 *
25	12%	17.44	14.71	15.10	1.00 *	0.99 *
30	12%	14.86	11.26	13.88	1.00 *	0.99 *

* = significantly different from 0 at " < 0.0001

proportion of retirement assets invested in equities versus debt. On the other hand, for high withdrawal rates (11% or 12%), the optimum asset allocation is 100% equity, but success rates generally fall below 50% even at the optimum 100% equity allocation.

For withdrawal rates of 6% and 7% our evidence is ambiguous. Maximum success for these two rates occurred at relatively low equity allocations ranging from 32% to 77%. Furthermore, the strength of the relationship between success and the percentage invested in equities is open to question. As financial advisors often recommend withdrawal rates in this range to their clients, we are somewhat disheartened at the weakness of our results across this range.

However, our results for withdrawal rates between 8% and 10% are unequivocal. For these withdrawal rates, the optimum asset allocation approaches 100% equities and there is a high correlation between success rates and the proportion of the portfolio invested in equities.

We conclude that asset allocation is unimportant at low withdrawal rates and pointless at high withdrawal rates (unless clients are willing to accept success rates below 50%). For withdrawal rates of from 8% to 10%, asset allocation is the crucial determinant of success and the optimum allocation is at or near to 100% equity.

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