Noise trading and stock market bubbles: what the derivatives market is telling us

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Abstract

Purpose – The authors examine the relation between noise trading in equity markets and stochastic volatility by estimating a two-factor jump diffusion model. Their analysis shows that contemporaneous price deviations in the derivatives market are statistically significant in explaining movements in index futures prices and option-market volatility measures.

Design/methodology/approach – To understand the impact noise may have in the S&P 500 derivatives market, the authors first measure and evaluate the influence noise exerts on futures prices and then investigate its influence on option volatility.

Findings – In the period from 1996 to 2003, this study finds significant changes in the volatility and mean reversion in the noise level and a significant increase in its relation to implied volatility in option prices. The results are consistent with a bubble in technology stocks that occurred with significant increases in noise trading.

Research limitations/implications – This study provides estimates for this model during the periods preceding and during the technology bubble. The study analysis shows that the volatility and mean reversion in the noise level are much stronger during the bubble period. Furthermore, the relation between noise trading and implied volatility in the futures market was of a significantly larger magnitude during this period. The study results support the importance of noise trading in market bubbles.

Practical implications – Bloomfield, O'Hara and Saar (2009) find that noise traders lower bid–ask spreads and improve liquidity through increases in trading volume and market depth. Such improved market conditions could have positive effects on market quality, and this impact could be evidenced by lower implied volatility when noise traders are more active. Indeed, the results in this study indicate that the level and characteristics of noise trading are fundamentally different during the technology bubble, and this noise trading activity has a larger impact during this period on implied volatility in the options market.

Originality/value – This paper uniquely analyzes derivatives on the S&P 500 Index in order to detect the presence and influence of noise traders. The authors derive and implement a two-factor jump diffusion noise model. In their model, noise rectifies the difference of analysts' opinions, market information and beliefs among traders. By incorporating a reduced-form temporal expression of heterogeneities among traders, the model is rich enough to capture salient time-series characteristics of equity prices (i.e. stochastic volatility and jumps). A singular feature of the authors' model is that stochastic volatility represents the random movements in asset prices that are attributed to nonmarket fundamentals.

Keywords Noise, Derivative markets, Bubbles

Paper type Research paper

1. Introduction

This paper analyzes derivatives on the S&P 500 Index in order to detect the presence and influence of noise traders. We derive and implement a two-factor jump diffusion noise model. In our model, noise rectifies the difference of analysts' opinions, market information and beliefs

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among traders. By incorporating a reduced-form temporal expression of heterogeneities among traders, the model is rich enough to capture salient time-series characteristics of equity prices (i.e. stochastic volatility and jumps). A singular feature of our model is that stochastic volatility represents the random movements in asset prices that are attributed to nonmarket fundamentals.

We assume that the market is composed of both rational and boundedly rational investors, who trade in equity markets. Efficient markets theory does not preclude the existence of boundedly rational agents, rather, it conjectures that a sufficiently large number of well-informed and well-financed investors (arbitrageurs) guarantees that any potential mispricing induced by noise traders will be corrected (Samuelson, 1965 and Fama, 1965). Despite the conviction of these researchers regarding the efficient market hypothesis, research suggests that, at times, arbitrageurs are unable or unwilling to remove noise traders from the market (Black, 1986; Barber *et al.*, 2009; Brunnermeier and Nagel, 2004). That is, some investors may interpret non-news price shocks as news itself and enact trading strategies on these non-news events (Roll, 1988). As such, investors face the inherent problem of determining what is "fundamental." (DeLong *et al.* 1990a, b).

Hence, the presence of noise traders can create externalities for investors by exposing them to additional risks that they fail to identify. A typical investor believes that asset prices should reflect their intrinsic value in a long-run equilibrium with short-term random shocks occurring due to fundamental information flow. Fundamentals, however, may not prevent anomalous price movements from occurring if differences in opinions among investors exist. That is, some investors may interpret non-news price shocks as news itself and enact trading strategies on these non-news events (Roll, 1988). As such, investors face the inherent problem of determining what is "fundamental."

In equilibrium, the price of any asset should equal the discounted expected cash flow and reflect a reasonable rate of return associated with its fundamentals. Clearly defining market fundamentals for equity has proven difficult. Standard discounted cash flow models require a discount rate estimate. This is further complicated by the fact that no definitive time horizon exists. Derivative securities, on the other hand, circumvent these issues. Recent advances in modern asset-pricing theory allow us to price a wide array of contingent claims, given a continuous-time model for the dynamics of the underlying state variables. The use of such models is born from a need to describe a range of economic states. Of particular interest trading patterns in equity markets can cause return distributions for these securities to be negatively skewed with higher kurtosis than allowable in a traditional lognormal model. The significant advantage of using the derivative market is that a clear distinction can be made between market fundamentals and noise.

In the next section, we review the option pricing literature. Section 3 develops the pricing model used to examine noise trading activity before and during the technology bubble in the late 1990s. Next we describe the data used in our analysis. Section 5 discusses the empirical results on implied volatility and noise trading. In the final section we summarize how noise trading and implied volatility changed during the tech bubble.

2. Literature review

Since the seminal work on option pricing by Black and Scholes (1972, 1973) and Merton (1973), many papers have provided insights on which of the basic assumptions lead to the greatest pricing inaccuracies. One issue with model accuracy is the "volatility smile." Specifically, the volatility implied by the option prices is dependent upon time to maturity and the extent to which an option is in the money, that is, "moneyness." Although Hull and White (1987), and later Heston (1993), provide a tractable stochastic volatility model, many studies show that stochastic volatility alone proves insufficient to account for the implied-pricing distributions found in the option data. In fact, numerous studies note that the volatility smile continues to persist, and thus, it appears that stochastic volatility models alone cannot correct for the volatility smile.

Bates (1988) first looks at the issues of using stochastic volatility without the potential for jumps and suggests that jumps are also needed. Pricing errors are associated with the assumptions of a lognormal distribution of stock prices and continuous returns. Furthermore, the empirical research typically supports including price jumps in option pricing models. For instance, Shiekh and Ronn (1994) analyze the patterns in option returns and find that not all patterns in adjusted volatility can be accounted for by stochastic volatility. Jackwerth and Rubinstein (1996) show the critical errors that result from a lognormal assumption and show how to uncover more dependable probability distributions from option prices. Bakshi *et al.* (1997) perform an exhaustive study looking at several variations of the Black–Scholes model. Furthermore, Bakshi *et al.* (1997) find empirical support for the stochastic volatility model, but they show that jumps are critical near an option's expiration. Duffie *et al.* (2000) show that models including a jump are superior to those with only stochastic volatility. Jackwerth (2000) gives a more detailed review of the literature on implied distributions of option prices and lends additional support to the notion that jumps matter. In a more comprehensive study, Jones (2000) thoroughly disclaims the single-parameter models by showing that volatility alone cannot adjust to compensate for the volatility smile.

In the pricing of options on futures, Ramaswamy and Sundaresan (1985) show notable estimation improvement with stochastic interest rates. Miltersen and Schwartz (1998) emphasize the importance of both stochastic interest rates and stochastic convenience yields. Hilliard and Reis (1998) further illustrate that extension of the model to include jumps in spot prices is important when valuing the options on commodity futures.

Lo and Wang (1995) forward a model with mean reversion and illustrate its importance in option pricing models. Schwartz and Smith (2000) develop a two-factor model (without jumps) that allows for mean reversion and uncertainty in equilibrium price level. They show that this model is equivalent to the stochastic convenience yield model. Lucia and Schwartz (2002), Escribano *et al.* (2002) and Deng (2001) examine the importance of the regular patterns (in the behavior of electricity prices) and detail its implications for the purposes of derivative pricing. Chacko and Das (2002) connect the relationship between affine stochastic processes and bond-pricing equations in exponential-term structure models. They forward numerous options that can use this method.

Considerable empirical evidence suggests that the majority of all financial assets, such as equities or equity indexes, currencies or interest rates, do not follow a lognormal random walk. One of the most prominent characteristics of the financial markets is that from time to time there is an abrupt, yet very significant, unpredicted change in an asset's price. These sudden moves occur far more frequently than would be predicted with the assumption of a lognormal distribution. Hence, it is likely that jump and bubble risk are priced.

3. Developing the pricing model

Empirical evidence illustrates that random movements in asset prices are attributed to fundamental information flow and noise. Moreover, current option research shows that stochastic volatility and jumps are necessary additions to the traditional Black–Scholes method to accurately describe the price dynamics of equity, that is, Hull and White (1987); Scott (1987); Heston (1993); Merton (1976); Bates (1991, 1996); Bakshi *et al.* (1997); Bakshi and Chen (1997) and Duffie *et al.* (2000). To model equity index prices, we assume the underlying price process for an equity index and its components are given by the following stochastic structure:

$$\frac{dS(t)}{S(t)} = \left(r - \delta + b(t) - \lambda^* \mu_J^*\right)dt + \sigma_s dZ_s^*(t) + J^* dq^*(t), \quad (1)$$

$$db(t) = -\kappa b(t)dt + \sigma_b dZ_b(t), \quad (2)$$

where r is the instantaneous riskless rate of return, λ is the frequency of jumps per year, δ represents the dividend yield, σ_s represents the variation coefficient for the stock process, $b(t)$ represents the presence of noise (or stochastic volatility) in the price of the market index, $Z_s(t)$ and $Z_b(t)$ are standard Brownian motions, and $Cov_t^*[dZ_s^*(t), dZ_b^*(t)] = \rho_{sb}dt$, $J^* = Y-1$ is the risk-adjusted percentage jump conditional upon a Poisson distributed event occurring that is log-normally, identically and independently distributed over time with unconditional mean μ_J^*. The standard deviation of $\ln(1+J)$ is ω, k , σ_b are respectively the speed of adjustment and variation coefficients for the noise diffusion $b(t)$, $q^*(t)$ is an independent Poisson jump counter with intensity λ, that is, $\Pr(dq^* = 1) = \lambda^* dt$, $q(t)$ and J are uncorrelated with each other and with $Z_s(t)$ and $Z_b(t)$.

The resulting sample path for equity prices stated in equation (1) will be continuous most of the time with finite jumps occurring at random times. Moreover, the stock return distribution in (1) is consistent with Bakshi *et al.* (1997) and offers a sufficiently versatile structure that accommodates the desired characteristics. That is, skewness in the return distribution is controlled by the correlation, ρ, or the mean jump, μ_J^*, whereas the amount of kurtosis is regulated by either the volatility parameter, σ_b, or the magnitude and variability of the jump component.

3.1 Futures valuation

If equation (1) conforms to some regularity conditions, then the random variable ratio of the index's price at time T to the index's price at time t can be written formally as

$$\frac{S(T)}{S(t)} = \exp\left\{\left(\left(r - \frac{1}{2}\sigma_s^2 - \delta\right) - \lambda^*\mu_J^*\right)\tau + \int_t^T b(v)dv + \sigma_s \int_t^T dZ_s^*(v)\right\} Y(n) \quad (3)$$

where $Y(n) = 1$ if $n = 0$; $Y(n) = \prod_{j=1}^{n} Y_j$ for $n \geq 1$. The Y_j are independently and identically distributed and n is Poisson distributed with parameter $\lambda^*\tau$, where $\tau = T - t$. If Y_j is log-normally distributed with mean $\left(\gamma - \frac{1}{2}\eta^2\right)$ and a variance of η^2, then the solution for the futures price according to the Feynman–Kac theorem is determined as follows: [1]

$$F(S(t), b(t), \tau) = S(t)A(\tau)e^{(r-\delta)\tau + H_b(\tau)b(t)} \quad (4)$$

where

$$A(\tau) = \exp\left(\frac{(H_b(\tau) - \tau)\left(\kappa\lambda\sigma_b - \frac{\sigma_b^2}{2} - \rho_{bs}\sigma_s\sigma_b\kappa\right)}{\kappa^2} - \frac{\sigma_b^2 H_b^2(\tau)}{4\kappa}\right)$$

$$H_b(\tau) = \frac{1 - e^{-\kappa\tau}}{\kappa}.$$

The futures valuation model in Eqn (4) has several distinct features. First, it shows that the futures price depends on the current level of the index price, the current level of noise, the time to maturity, the parameters of the joint process and the price of a risk-free, zero-coupon bond with the same corresponding maturity. Moreover, (4) specifies an exact relation between the futures price and both market fundamentals and the level of noise in the equity market. In particular, we note that in the absence of noise ($\sigma_b = 0$) [2], (4) reduces to the traditional costof-carry model. This illustrates that in equilibrium index futures are indeed priced according to their fundamentals. Furthermore, we find that as maturity diminishes, the impact of noise on futures prices is decreasing. This is expected since the futures price must converge to the spot price at expiry. Lastly, when the intensity of the noise to revert to the mean increases, its influence on futures diminishes.

The closed-form formula in Eqn (4) makes it possible to measure the amount of volatility present in the market. The logarithm of the futures price is linear with respect to the spot price and noise. The level of noise is latent, but its relation to the futures price is known. Using the Kalman filter, point estimates for the amount of noise present in equity prices can be obtained using the pricing relation in Eqn (4). Once the estimates are obtained, they may be used to test their significance to explain futures prices.

3.2 Option valuation

To price an option contract standard practice begins with specifying a stochastic structure governing the underlying state variables. Uniquely, options written on index futures are not written directly on the index and therefore are not directly influenced by the index's stochastic structure. The value of these securities is only influenced through the price movements of the *futures* contracts. Therefore, to price the futures options, the stochastic composition of the futures contract must be developed endogenously from the underlying system of state variables that impact the value of the futures contract.

The futures price is a function of the spot price, convenience yield and time. Using Ito's lemma, and the dynamics in equations (1) and (2), the stochastic increments for the futures contract are shown to be

$$\begin{aligned} dF(t) = {} & \left\{ \frac{1}{2} F_{ss}\sigma_s^2 S^2 + \frac{1}{2} F_{bb}\sigma_b^2 + F_s\left(r - \delta + b(t) - \lambda^* \mu_J^*\right)S - F_b \kappa b(t) + F_{sb}\rho_{sb}\sigma_s\sigma_b + F_t \right. \\ & \left. + \lambda^* E^*[F(SY, t) - F(S, t)] \right\} dt + F_s\sigma_s S dZ_s^*(t) + F_b\sigma_b dZ_b^*(t) + F(SY, t) \\ & - F(S, t) - \lambda^* E^*[F(SY, t) - F(S, t)]dt. \end{aligned} \tag{5}$$

The drift coefficient in equation (5) is equal to the equilibrium condition for the valuation model in equation (4), thereby reducing (5) to

$$dF(t) = F_s\sigma_s S dZ_s^*(t) + F_b\sigma_b dZ_b^*(t) + F(SY, t) - F(S, t) - \lambda^* E[F(SY, t) - F(S, t)]dt. \tag{6}$$

As expected, continuous movements in the futures price are attributed to the diffusions for the spot price and noise processes, while discrete movements occur due to random jumps in the spot market. A unique feature of the futures diffusion is that jumps are introduced as a function of the futures price, which differs from previous research (see Hilliard and Reis (1998)). Previous research shows that jumps from the spot market are identical to jumps in the futures contract indicating a one-to-one relation. Our model shows that jumps in the spot market do impact the dynamics of the futures price but not necessarily in the same manner. Moreover, jumps across contracts could differ, which could explain pricing differences in option contracts written on these contracts.

3.3 Taking the partial derivatives

$$F_s = A(\tau)e^{(r-\delta)\tau + H_b(\tau)b(t)}.$$

$$F_b = H_b(\tau)F(SY,\, b,\, \tau).$$

and substituting them into Eqn (6) yields

$$\frac{dF(t)}{F(t)} = -\lambda^* \mu_F^* dt + \sigma_s dZ_s^*(t) + H_b(\tau)\sigma_b dZ_b^*(t) + J_F^* dq_F^*. \tag{7}$$

Defining the diffusion for the futures to be

$$\sigma_F dZ_F^*(t) \equiv \sigma_s dZ_s^*(t) + H_b(\tau)\sigma_b dZ_b^*(t), \tag{8}$$

the futures price stochastic differential becomes

$$\frac{dF(t)}{F(t)} = -\lambda^* \mu_F^* dt + \sigma_F dZ_F^*(t) + J_F^* dq_F^*. \tag{9}$$

The stochastic differential in expression (9) is for an equity index futures contract, wherein the value derives from a jump-diffusion spot price and a mean-reverting noise process. This expression is determined endogenously from the dynamics of the state variables, which are given exogenously. In addition, the diffusion is independent of the level of both state variables. Therefore, for the purpose of pricing options written on the underlying futures contract, the futures price dynamic can be used as an exogenous process.

The solution for the two-factor options price is given as

$$C(FY,\, t) = e^{-r\tau_1} \sum_{n=0}^{\infty} \frac{e^{-\lambda^*\tau_1}\left(\lambda^*\tau_1\right)^n}{n!} E_t^*\left[\max[F(T) - X,\, 0]\right], \tag{10}$$

$$C(FY,\, t) = e^{-r\tau_1} \sum_{n=0}^{\infty} \frac{e^{-\lambda^*\tau_1}\left(\lambda^*\tau_1\right)^n}{n!} \left[F(t)e^{g(n)\tau_1} N(d_1) - XN(d_2)\right], \tag{11}$$

where

$$g(n) = \left(-\lambda^* \mu_F^* + \frac{n\gamma_F^*}{\tau_1}\right),$$

$$v^2 = V_t^*\left[\int_t^{T_1} \sigma_F dZ^*(s)\right],$$

$$d_1 = \frac{\ln\left(\frac{F(t)}{X}\right) + \left(\left(-\lambda^* \mu_F^* + \frac{n\gamma_F^*}{\tau_1}\right)\tau_1 + \frac{1}{2}\left(v^2 + n\omega^2\right)\right)}{\sqrt{v^2 + n\omega^2}},$$

and

$$d_2 = d_1 - \sqrt{v^2 + n\omega^2},$$

The solution for the option contract is consistent with those presented by Merton (1976) and Bates (1991, 1996).

4. Data

Our analysis uses weekly observations for spot, futures and option prices for the S&P 500 Index. To estimate our model, we use the three closest to maturity futures contracts on the S&P 500 Index. End-of-day futures prices are obtained from Price-data.com for the period from May 1982 to May 2003. When a contract is within five days of its expiration, we use the price of the subsequent futures contract and continue until this contract is five days from its expiration. In addition, data for the three-month T-bill is obtained from the Federal Reserve's H.15 Statistical Release and data used for the S&P 500 Index dividend yield are taken from Datastream.

Options data for this study is obtained from the Chicago Mercantile Exchange (CME) over the period from January 1983 to May 2003. This data is similar to the quotes capture report used in Bates (1991). However, our focus is over a longer period of time and does not include an analysis of intraday effects. We use daily settlement prices for S&P 500 futures options. Price Files are produced in the late afternoon of each business day after all CME markets are closed. The options are on a cash settled with the underlying contract being $500 times the S&P 500 Index level. Contracts are available on five-point intervals above and below the current index level. Thus, as with other option contracts, the range of strike prices is determined by past price movements of the underlying asset.

In this study we use three filters on the data. First we study contracts with a single maturity; only contracts with 1–4 months are considered. This avoids thin trading and expiration effects. Next, we require that all contracts have 20 trades in calls and 20 trades in puts. Again this restriction is to avoid thin trading. Finally, we restrict analysis to contracts that have at least four different strike ranges for calls and puts. This restriction insures a reasonable moneyness range to study.

5. Empirical analysis

To understand the impact noise may have in the S&P 500 derivatives market, we first measure and evaluate the influence noise exerts on futures prices and then investigate its influence on option volatility.

5.1 Futures

Equation (4) illustrates the hypothesized relation between the S&P 500 Index, noise and futures written on the Index. A unique element of this relation is that the noise process is latent. Therefore, to econometrically evaluate our model, we employ dynamic factor analysis to document the time-series behavior of noise and to test its statistical significance for pricing index futures.

An instrument capable of handling a wide variety of time-series models, including latent variable estimation, is the state-space model (see Watson and Engle, 1983). The general state-space form applies to a multivariate time series of observable variables. In this case, futures prices for different maturities are related to an unobservable state vector, and a mean-reverting stochastic noise process is included via a measurement equation. The elements of the noise process are determined by the state or transition equation, which in our model is the discrete time version of (2). To configure the state-space structure for our futures market model, let $b(t)$ be an $n \times 1$ vector of unobserved variables called noise, and $x(t)$ and $y(t)$ be m x1 and qx1 vectors of observable variables called input and output variables, respectively. The state-space model can be written as:

$$b(t+1) = Gb(t) + \xi(t+1), \tag{12a}$$

$$y(t) = C + Dx(t) + Mb(t) + \zeta(t), \tag{12b}$$

where $\xi(t)$ and $\zeta(t)$ are, respectively, nx1 and qx1 vectors of disturbances with G, C, D and M as constant real vectors of conformable dimensions. More formally

$$y(t) = [\ln F(S(t),\, \tau_i)] \quad i = 1,\ \ldots,\ q,$$

$$x(t) = [(r - \delta),\, \ln S(t)],\ G = [1 - \kappa \Delta t],$$

$$C = [\ln A(\tau_i)],\ D = [\tau_i,\, 1],$$

$$M = [H_b(\tau_i)],$$

We assume that the disturbance terms, $\xi(t)$ and $\zeta(t)$, are each serially uncorrelated and are also uncorrelated with each other and that

$$E(\xi(t)) = 0,\ E(\zeta(t)) = 0,$$

$$E\left(\xi(t)\xi'(t)\right) = \sigma^2 \Delta t,\ E\left(\zeta(t)\zeta'(t)\right) = R.$$

Given the structure of the state space, the Kalman filter may be used to generate conditional forecasts of the noise process [3]. This recursive procedure produces optimal estimators of the noise at time t, given information up to time t, and it enables the estimate of this latent process to be continuously updated as new information becomes available. Operationally, the best linear estimate of the noise process, $\hat{b}(t|t)$, is obtained through the following equations:

$$\hat{b}(t+1|t) = G\hat{b}(t|t), \tag{13a}$$

$$P(t+1|t) = GP(t|t)G' + V, \tag{13b}$$

$$\hat{\zeta}(t+1|t) = y(t+1) - G\hat{b}(t+1) - Dx(t+1), \tag{13c}$$

$$K(t+1) = P(t+1|t)G'\left[GP(t+1|t)G' + R\right]^{-1}, \tag{13d}$$

$$\hat{b}(t+1|t+1) = \hat{b}(t+1|t) + K(t+1)\zeta(t+1|t), \tag{13e}$$

$$P(t+1|t+1) = [I - K(t+1)G]P(t+1|t), \tag{13f}$$

where

$$P(t+1|t) \equiv E\left[\left(b(t+1) - \hat{b}(t+1|t)\right)\left(b(t+1) - \hat{b}(t+1|t)\right)'\right],$$

and

$$P(t+1|t+1) \equiv E\left[\left(b(t+1) - \hat{b}(t+1|t+1)\right)\left(b(t+1) - \hat{b}(t+1|t+1)\right)'\right],$$

are the error covariance matrices, and $1 \le t \le T$.

Lastly, the Kalman filter treats the model's parameters as known. In practice, however, the parameter matrices C, D, G, M, $\sigma^2 \Delta t$ and R are unknown and need to be estimated. To begin estimating the parameters, initial guesses are presumed and used to produce first-generation forecasts of the state vector's time series. Once the noise estimates have been computed, the data are used to maximize a log likelihood function to acquire the parameters for the model. These estimated parameters may then be recycled in the Kalman filter to reproduce updated estimates of the state vector, which leads to parameter estimates that ultimately produce the greatest likelihood value. After this iterative procedure converges, final parameter and noise estimates are obtained, and their significance is statistically tested.

Iterating between the maximum likelihood step to estimate the parameters of (12) and the Kalman step to generate the unobservable level of noise, dynamic factor analysis concludes that both the spot price and noise process are statistically significant in determining futures prices. More specifically, Table 1 reports the statistical importance of noise in the determination of the S&P 500 futures prices. For our analysis, we estimate our model over four different time periods using the spot and futures prices. Column 1 reports the results for our model over the entire sample period, which is from 1982 to 2003. This period starts when the CME introduced trading in S&P 500 futures contracts in 1982. Our sample period ends in 2003 with the conclusion of the bear market in stocks after the technology bubble.

Column 2 reports results for our model using data from 1988 to 2003. We choose this period for analysis as it is subsequent to two events that might influence our findings. First, Merrick (1988) identifies pricing anomalies that existed with the S&P 500 futures contract from 1982 to 1985, when the efficiency of arbitrage in the Index improved. Second, the stock market crash of 1987 resulted in significant volatility in the stock market. Lastly, columns 3 and 4 report results for the time periods before and during the hypothesized technology bubble [4]. That is, we estimate our model from 1988 to 1996 and from 1996 to 2003.

In all cases, the speed of adjustment coefficient (k), the diffusion coefficient (α) and the market price of risk (λ) are all statistically significant at the 0.01 level. Comparing the results for the first three columns, we find there is little difference between the coefficients for the noise process. The main difference between the first estimation period and the second estimation period is that the former has a moderately higher tendency of mean reversion. This result may be caused by pricing anomalies that existed with the S&P 500 futures contract from 1982 to 1985 as documented by Merrick (1988) [5]. Lastly, we note that the volatility for the noise process is small and the market price of risk is close to zero.

Comparing our results for the bubble period relative to the pre-bubble period, we find that the volatility and mean reversion in the noise level are much stronger during the bubble period. The estimated coefficient for the speed of adjustment (k) is 1131.6 during the tech bubble. This is twice as large as the coefficients for the entire sample period (578.4) and pre-tech bubble (426.6). Furthermore, the estimated coefficient for the variation of the noise diffusion ($b(t)$) is 1145.9, which is significantly larger than for other estimation periods and 2.6 times larger than the coefficient for the pre-tech bubble period.

	Estimation periods							
Model estimates	Entire sample 04/1982–05/2003		Post-1987 crash 01/1988–05/2003		Pre-tech bubble 01/1988–12/1996		Tech bubble 12/1996–05/2003	
κ	578.4	(403.4)	720.3	(384.1)	426.6	(118.8)	1131.6	(209.3)
σ_B	585.3	(409.4)	728.8	(390.6)	431.0	(119.8)	1145.9	(217.3)
$\rho\sigma_S$	590.3	(413.9)	735.0	(395.4)	434.1	(120.6)	1156.3	(223.2)
λ	0.112	(0.164)	0.098	(0.160)	0.131	(0.125)	0.056	(0.198)
ε_1	0.058	(0.023)	0.050	(0.019)	0.056	(0.018)	0.041	(0.016)
ε_2	0.029	(0.012)	0.024	(0.009)	0.030	(0.005)	0.015	(0.003)

Note(s): This table shows estimation results for the two-factor model using the Kalman filter. The data used in the estimation are the weekly spot level of the S&P 500 Index, the second and third closest to expiration futures contracts on the S&P 500 Index, the three-month T-bill rate and the dividend yield of the S&P 500 Index. Standard errors are shown in parentheses

Table 1. Estimation of the two-factor model for the S&P 500 Index

These findings are consistent with observations of other researchers who suggest a bubble existed during this time period. In particular, Brunnermeier and Nagel (2004) find that hedge funds were holding a disproportionately high level of technology stocks during this time. These portfolio holdings peeked six months prior to 2000 and were not offset by holdings in the derivatives market. These hedge fund portfolios would not necessarily create high volatility, but Abreu and Brunnermeier (2003) and DeLong *et al.* (1990b) show that well-informed agents find it optimal to follow boundedly rational trading strategies to create mispricing in the market.

Barber *et al.* (2009) show that the aggregate trading patterns of individual investors, who are drawn to stocks with strong past returns and concentrate their trading in select stocks, do influence asset prices. In their examination of large discount changes in closed-end funds, Hughen and McDonald (2005) document that institutional investors can also cause prices to deviate from intrinsic values. Thus during this time period, it is possible to conclude that trading strategies enacted by hedge funds may have initiated additional trading patterns from noise traders. Our findings of significantly higher mean reversion and volatility suggest the market did witness an increase in uncertainty due to the presence of noise traders.

5.2 Options

The empirical evidence earlier suggests that noise is present in the equity market, which causes equity prices to exhibit periods of stochastic volatility. A natural result from the pricing relation in Eqn (11) shows that these random movements in volatility should influence the futures option market. To understand the impact of noise traders on the options written on index futures, we analyze the volatility structure of the options market. To do this, we back out a Black–Scholes (BS)-implied volatility from each option price in the sample. Then, we equally weight the implied volatilities of all call options in a given moneyness–maturity category, to produce separate average implied volatility for the futures options.

Table 2 reports the average BS volatility values across five moneyness and three maturity categories, for both the entire sample period and subperiods defined in Table 1. Regardless of the estimation period or term to expiration, the BS-implied volatilities exhibit a strong smile pattern as the options go from deep in the money to deep out of the money. As an example of this relation, consider the implied volatilities for the nearby futures contract over the entire sample from 1982 to 2003. The deep in-the-money call options, which have a moneyness less than or equal to 0.97, have an average implied volatility estimate of 0.1667, and this is significantly less than the average implied volatility estimate of 0.2041 for the deep out-of-the-money options. Furthermore, the volatility smiles are strongest for short-term options. These findings of moneyness-related and maturity-related biases associated with the BS model are consistent with those in the existing literature, for example, Bakshi *et al.* (1997); Bates (1996).

According to the aforementioned futures valuation model, the maturity-related biases exhibited by BS-implied volatilities are not completely unanticipated. In particular, the futures price volatility and therefore option volatility should decrease monotonically with respect to the term to expiration. Under the stochastic structure of the equity market, stochastic increments in the futures prices, Eqns (7) and (8), are exponentially dampened with respect to maturity. Intuitively, noise trading leads asset prices to deviate from their fundamental values, which after some point in time induces arbitrageurs to become more active in the market. As a result, the impulse from random price movement due to noise should diminish over time. For deep in the money, at the money and out of the money, implied volatilities exhibit this behavior.

In addition to the term structure of volatility, structural shifts in the BS-implied volatilities seem to exist. In particular, consistent with our empirical observation that greater levels of noise existed during the technology bubble. In particular, implied volatilities across moneyness and maturity categories are 41–56% higher during the technology bubble period

Period	Moneyness	Nearby	Contract 2nd out	3rd out
1982–2003	$0.95 < m \leq 0.97$	0.1667	0.1613	0.1624
	$0.97 < m \leq 0.99$	0.1691	0.1661	0.1678
	$0.99 < m \leq 1.01$	0.1786	0.1734	0.1727
	$1.01 < m \leq 1.03$	0.1911	0.1821	0.1793
	$1.03 < m \leq 1.05$	0.2041	0.1915	0.1889
1988–2003	$0.95 < m \leq 0.97$	0.1658	0.1613	0.1618
	$0.97 < m \leq 0.99$	0.1698	0.1685	0.1684
	$0.99 < m \leq 1.01$	0.1810	0.1774	0.1751
	$1.01 < m \leq 1.03$	0.1964	0.1875	0.1823
	$1.03 < m \leq 1.05$	0.2139	0.1994	0.1922
1988–1996	$0.95 < m \leq 0.97$	0.1395	0.1321	0.1353
	$0.97 < m \leq 0.99$	0.1400	0.1389	0.1418
	$0.99 < m \leq 1.01$	0.1497	0.1476	0.1489
	$1.01 < m \leq 1.03$	0.1649	0.1580	0.1576
	$1.03 < m \leq 1.05$	0.1836	0.1682	0.1675
1996–2003	$0.95 < m \leq 0.97$	0.2054	0.2062	0.2068
	$0.97 < m \leq 0.99$	0.2154	0.2145	0.2140
	$0.99 < m \leq 1.01$	0.2290	0.2239	0.2210
	$1.01 < m \leq 1.03$	0.2449	0.2330	0.2267
	$1.03 < m \leq 1.05$	0.2602	0.2423	0.2357

Note(s): This table provides the implied volatility estimates from the S&P 500 futures call options. For each time frame, the futures options are segmented by moneyness (futures price/strike price) and maturity (contract expiration)

Table 2. Implied volatility estimates

relative to the pre-bubble period. Consider the category that includes at-the-money options (moneyness between 0.99 and 1.01) for the nearby futures contract. These calls have an average implied volatility of 0.2290 for the tech bubble period, and this result is 53% higher than the implied volatility estimate of 0.1497 for the period from 1988 to 1996. Furthermore, this volatility estimate is 28% higher than the corresponding estimate for the entire sample period. The marked difference in volatility levels across these time periods seems to support our findings that the equity market witnessed a structural change in trading during the late 1990s.

5.3 Noise and implied volatility

To understand the volatility structure observed in the index futures options market, we employ regression analysis to determine the association between noise present in the equity market and the level of implied volatility in option prices. The simple regression specification is:

$$\mathrm{IV}_{i,j,t} = \alpha_{i,j} + \beta_{i,j}\mathrm{Noise}_t + \varepsilon_{i,j,t}, \tag{14}$$

where $\mathrm{IV}_{i,j,t}$ is the average implied volatility for moneyness and maturity category over the sample (sub)period, and Noise_t, as measured by the Kalman filter, represents the current level of stochastic volatility that exists in the equity market. Eqn (14) tests the association of the market's expectation of future risk relative to contemporary deviations in the market.

Table 3 provides the estimated regression coefficients over our entire sample period for options in five categories of moneyness. Panel A shows the results for the futures contract with the closest expiration, and Panels B and C provide the results for the next two futures contracts to expire. The regression coefficient for the noise variable ($\beta_{i,j}$) is negative and

	Moneyness 0.95–0.97	0.97–0.99	0.99–1.01	1.01–1.03	1.03–1.05
Panel A: First futures contract out (nearby)					
$\alpha_{i,j}$ (intercept)	0.1698 (0.0020)	0.1724 (0.0022)	0.1817 (0.0022)	0.1954 (0.0023)	0.2084 (0.0024)
$\beta_{i,j}$ (noise)	−0.7457 (0.2119)	−0.7953 (0.2284)	−0.8103 (0.2331)	−1.0424 (0.2471)	−1.0203 (0.2456)
Adj R^2	0.0118	0.0113	0.0113	0.0175	0.0174
N	952	975	966	944	920
Panel B: Second futures contract out					
$\alpha_{i,j}$ (intercept)	0.1651 (0.0020)	0.1693 (0.0020)	0.1769 (0.0020)	0.1862 (0.0022)	0.1940 (0.0023)
$\beta_{i,j}$ (noise)	−0.8749 (0.2018)	−0.7643 (0.1985)	−0.8271 (0.2088)	−0.9893 (0.2400)	−0.6735 (0.2385)
Adj R^2	0.0183	0.0145	0.0156	0.0178	0.0094
N	954	939	931	883	740
Panel C: Third futures contract out					
$\alpha_{i,j}$ (intercept)	0.1658 (0.0022)	0.1702 (0.0021)	0.1763 (0.0022)	0.1830 (0.0024)	0.1939 (0.0028)
$\beta_{i,j}$ (noise)	−0.8942 (0.2809)	−0.6315 (0.2608)	−0.8678 (0.2575)	−0.9081 (0.2882)	−1.2213 (0.3691)
Adj R^2	0.0029	0.0062	0.0132	0.0134	0.0182
N	775	775	776	656	538

Note(s): This table provides the results from the regression $IV_{i,j},t = \alpha_{i,j} + \beta_{i,j}$, Noise $t + \varepsilon_{i,j,t}$, where i = contract maturity, j = moneyness and IV = average implied volatility. Observations with less than two days to expiration, IV = 0 or moneyness <0.95 or >1.05 are deleted from the sample. Standard errors are shown in parentheses

Table 3.
Regressions of implied volatility upon noise, full sample period

statistically significant at the 1% level for all models in the three panels. In other words, the measure of noise derived from the Kalman filter is inversely related to the expected volatility for the S&P 500 Index. This finding is robust to the moneyness of the option and expiration date of the futures contract used in the analysis.

We next examine whether the relation between implied volatility and noise is stable in the periods before and during the technology bubble. Table 4 provides the output from a regression analysis over the pre-tech bubble period, which we define as from 1988 to 1996. The estimated coefficients for the noise variable are of primary interest to our study, and our analysis shows that noise is negative and statistically significant at the 1% level for all the model specifications and Panels A, B and C. The coefficient estimates in Table 4 are larger in absolute value terms than the estimates shown in Table 3. For example, Panel A of Table 4 shows that the noise coefficient for the at-the-money category of call options is −1.1083, which is 37% larger than the corresponding estimate in Table III (−0.8103).

Table 5 provides the regression output for the period associated with the technology bubble, which is from 1996 to 2003. The sign and significance of the regression coefficients for noise are similar to the results for both of the full sample period and the pre-tech bubble period. However, the magnitude of the coefficients estimated using the tech bubble period is fundamentally larger than that of the coefficients from the other periods. Consider the estimated coefficient for the noise variable of −3.9075, which is shown in Panel A of Table 5. This coefficient has a magnitude that is 4.5 times the corresponding coefficient found in Table 4 for the pre-tech bubble period. In a comparison of Tables 4 and 5, the coefficient estimates for noise are on average 3.5 times larger in the models shown in Panel A and 2.5 times larger in the models shown in Panel B. In other words, the analysis supports a larger negative relation between noise and implied volatility during the period commonly associated with the technology bubble.

These results are consistent with prior research done on noise at the market microstructure level. Li (2016) investigates the impact of liquidity on volatility using

	Moneyness 0.95–0.97	0.97–0.99	0.99–1.01	1.01–1.03	1.03–1.05
Panel A: First futures contract out (nearby)					
$\alpha_{i,j}$ (intercept)	0.1518 (0.0021)	0.1519 (0.0022)	0.0159 (0.0022)	0.1709 (0.0022)	0.1832 (0.0021)
$\beta_{i,j}$ (noise)	−0.8766 (0.1956)	−0.9915 (0.2038)	−1.1083 (0.0240)	−1.3296 (0.2067)	−1.3296 (0.1974)
Adj R^2	0.0286	0.0326	0.0412	0.0590	0.0658
N	650	673	664	645	631
Panel B: Second futures contract out					
$\alpha_{i,j}$ (intercept)	0.1455 (0.0020)	0.1484 (0.0017)	0.1552 (0.0019)	0.1643 (0.0021)	0.1694 (0.0022)
$\beta_{i,j}$ (noise)	−1.0644 (0.1814)	−1.0521 (0.1676)	−1.1379 (0.1733)	−1.4400 (0.2058)	−1.1646 (0.1992)
Adj R^2	0.0482	0.0559	0.0614	0.0731	0.0632
N	661	650	645	609	493
Panel C: Third futures contract out					
$a_{i,j}$ (intercept)	0.1465 (0.0022)	0.1501 (0.0020)	0.1559 (0.0020)	0.1631 (0.0021)	0.1755 (0.0025)
$\beta_{i,j}$ (noise)	−1.7022 (0.2564)	−1.3834 (0.2222)	−1.4590 (0.2138)	−1.6311 (0.2424)	−2.4967 (0.9619)
Adj R^2	0.0758	0.0672	0.0788	0.0897	0.1434
N	526	525	534	450	370

Notes(s): This table provides the estimates from the following regression equation $IV_{i,j,t} = \alpha_{i,j} + \beta_{i,j}\,\text{Noise}_t + \varepsilon_{i,j,t}$, where i = contract maturity, j = moneyness and IV = average implied volatility. Observations with less than two days to expiration, IV = 0 or moneyness <0.95 or >1.05 are deleted from the sample. Standard errors are shown in parentheses.

Table 4. Regressions of implied volatility upon noise, pre-1996 period

NYSE stocks and index futures. His analysis isolates the noise component in liquidity trading using trading volume and commissions. Similar to the results presented in our study, Li finds that noise has a negative relation with both *ex ante* and *ex post* return volatility and this negative relation is higher during market crises. Our study confirms these findings through a different approach for quantifying noise trading through the index futures market.

	Moneyness 0.95–0.97	0.97–0.99	0.99–1.01	1.01–1.03	1.03–1.05
Panel A: First futures contract out (nearby)					
$\alpha_{i,j}$ (intercept)	0.2272 (0.0049)	0.2371 (0.0044)	0.2509 (0.0052)	0.2674 (0.0054)	0.2828 (0.0057)
$\beta_{i,j}$ (noise)	−3.9075 (0.7054)	−3.9073 (0.7361)	−3.9321 (0.7577)	−4.0852 (0.7849)	−4.0182 (0.8369)
Adj R^2	0.0898	0.0828	0.0793	0.0805	0.0711
N	302	302	302	299	289
Panel B: Second futures contract out					
$\alpha_{i,j}$ (intercept)	0.2234 (0.0043)	0.2303 (0.0045)	0.2409 (0.0046)	0.2486 (0.0047)	0.2584 (0.0050)
$\beta_{i,j}$ (noise)	−3.0182 (0.6143)	−2.7562 (0.6399)	−2.9611 (0.6528)	−2.7325 (0.6629)	−2.7878 (0.7207)
Adj R^2	0.0734	0.0574	0.0642	0.0553	0.0537
N	293	289	286	274	247
Panel C: Third futures contract out					
$\alpha_{i,j}$ (intercept)	0.2204 (0.0044)	0.2243 (0.0045)	0.2355 (0.0046)	0.2383 (0.0050)	0.2450 (0.0070)
$\beta_{i,j}$ (noise)	−2.3500 (0.6341)	−1.7806 (0.6407)	−2.4790 (0.6446)	−1.9688 (0.6900)	−1.4869 (0.9619)
Adj R^2	0.0488	0.0257	0.0541	0.0337	0.0083
N	249	250	242	206	168

Note(s): This table provides the estimates of the following regression equation: $IV_{i,j,t} = \alpha_{i,j} + \beta_{i,j}\,\text{Noise}_i + \varepsilon_{i,j,t}$, where i = contract maturity, j = moneyness and IV = average implied volatility. Observations with less than two days to expiration, IV = 0 or moneyness <0.95 or >1.05 are deleted from the sample. Panel A displays the nearby contract, Panel B the second nearest term contract and Panel C the longest-term contract. Standard errors are shown in parentheses

Table 5. Regressions of implied volatility upon noise, post-1996 period

Why would noise trading have a negative relation with volatility? An experimental study by Bloomfield *et al.* (2009) to examine the behavior of noise traders in a laboratory market offers insights into a potential explanation. They find that noise traders lower bid–ask spreads and improve liquidity through increases in trading volume and market depth. Such improved market conditions could have positive effects on market quality, and this impact could be evidenced by lower implied volatility when noise traders are more active. Indeed, the results in our study indicate that the level and characteristics of noise trading are fundamentally different during the technology bubble, and this noise trading activity has a larger impact during this period on implied volatility in the options market.

6. Conclusions

We estimate a two-factor jump diffusion noise model for futures contracts on the S&P 500 Index. This model provides insight into key pricing parameters, especially implied volatility and noise. Our study provides estimates for this model during the periods preceding and during the technology bubble. Our analysis shows that the volatility and mean reversion in the noise level are much stronger during the bubble period. Furthermore, the relation between noise trading and implied volatility in the futures market was of a significantly larger magnitude during this period. Our results support the importance of noise trading in market bubbles.

Notes

1. This solution is a stylized version of Hilliard and Reis (1998).
2. If the variance is zero, then the bubble is constant and the level of the bubble will equal its long-run average. In our model, this value is equal to zero.
3. The Kalman filter technique is well treated in the control literature and the interested reader is referred to Harvey (1989) or Hamilton (1991) for details.
4. During this period, the Nasdaq Composite Index increased by approximately 400%, and the technology bubble peaked in March 2000 with the Index trading at a price/earnings ratio of almost 200. The subsequent bear market in technology stocks ended in 2003.
5. Merrick (1988) identifies mispricing in S&P 500 futures contracts, and he documents that these contracts perform significantly better as hedges after 1985.
6. While the form of the stochastic differential is identical to the Hilliard and Reis (1998) model, the assumed parameter values are different. Hilliard and Reis presume that the Bates' (1991, 1996) model is applicable and define their model accordingly. We simply use a no arbitrage constraint.

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Appendix 1
Derivation of pricing model (Equation #3)

In this section, we present a complete derivation of pricing model (3) from the paper.

Recall the following assumptions are maintained:

A1. There are no arbitrage opportunities.

A2: Trading takes place continuously.

A3: There are no transaction costs, taxes and short sales constraints.

A4: The dynamics of the commodity prices are given by the following stochastic differential equation [6]

$$\frac{dS(t)}{S(t)} = \left(r - \delta + b(t) - \lambda^* \mu_J^*\right) dt + \sigma_s dZ_s^*(t) + J^* dq^*, \tag{1}$$

where r is the instantaneous riskless rate of return; δ represents the dividend; $b(t)$ represents the presence of noise in the price of the market index; σ_s represents the variance proportional to price changes; $dZ_s^*(t)$ is the risk-adjusted increment of a standard Brownian motion process; q^* is the independent Poisson process; $J^* = Y - 1$ is the risk-adjusted random percentage jump conditional upon a Poisson distributed event occurring; $\mu_J^* = E^*[J^*]$; and $\Pr(dq^* = 1) = \lambda^* dt$.

A5: The noise/bubble process follows the Ornstein–Uhlenbeck process

$$db(t) = -\kappa b(t)dt + \sigma_b dZ_b^*(t), \tag{2}$$

where k is the speed of adjustment factor; σ_b is the diffusion coefficient for the bubble process; and $dZ_b^*(t)$ is the risk-adjusted increment of a standard Brownian motion.

A6: The correlation between the Brownian motions is

$$\text{Cov}_t^*\left[dZ_s^*(t), dZ_b^*(t)\right] = \rho_{sb} dt. \tag{3}$$

A7: The Brownian motions and the Poisson process are uncorrelated.

A8: The instantaneous rate of return and the dividend yield are constant.

Two-factor jump-diffusion futures valuation

Given the commodity spot price dynamic of expression (1), it is seen that the resulting sample path will be continuous most of the time with finite jumps occurring. These jumps will arrive with different signs and amplitudes at discrete points in time. If expression (1) conforms to some regularity conditions, then the random variable ratio of the commodity price at time T to the commodity price at time t can be written formally. Let $X(t) = \text{In}\, S(t)$. Applying Ito's lemma to the transformation, the following dynamics for $X(t)$ may be written as

$$\begin{aligned} dX &= X_s dS_{dq=0} + \frac{1}{2} X_{ss} [dS_{dq=0}]^2 + XY - X \\ &= \frac{1}{S}\left(\left(r - \delta + b(t) - \lambda^* \mu_J^*\right) S dt + \sigma_s S dZ_s^*(t)\right) - \frac{1}{2S^2}\sigma_s^2 S^2 dt + \ln SY - \ln S \\ &= \frac{1}{S}\left(\left(r - \delta + b(t) - \lambda^* \mu_J^*\right) S dt + \sigma_s S dZ_s^*(t)\right) - \frac{1}{2S^2}\sigma_s^2 S^2 dt + \ln\left(\frac{SY}{S}\right) \\ &= \frac{1}{S}\left(\left(r - \delta + b(t) - \lambda^* \mu_J^*\right) S dt + \sigma_s S dZ_s^*(t)\right) - \frac{1}{2S^2}\sigma_s^2 S^2 dt + \ln Y \\ &= \left(r - \delta + b(t) - \lambda^* \mu_J^*\right) dt + \sigma_s dZ_s^*(t) - \frac{1}{2}\sigma_s^2 dt + \ln Y \\ &= \left(r - \frac{1}{2}\sigma_s^2 - \delta + b(t) - \lambda^* \mu_J^*\right) dt + \sigma_s dZ_s^*(t) + \ln Y, \end{aligned} \tag{4}$$

Integrating over the aforementioned expression from t to T yields the following:

$$\int_t^T dX(v) = \int_t^T \left(\left(r - \frac{1}{2}\sigma_s^2 - \delta\right) - \lambda^* \mu_J^*\right) dv + \int_t^T b(v) + \sigma_s \int_t^T dZ_s^*(v) + \sum_{j=1}^n \ln(Y_j),$$

$$X(T) - X(t) = \int_t^T \left(\left(r - \frac{1}{2}\sigma_s^2 - \delta\right) - \lambda^* \mu_J^*\right) dv + \int_t^T b(v)dv + \sigma_s \int_t^T dZ_s^*(v) + \sum_{j=1}^n \ln(Y_j),$$

$$X(T) = X(t) + \left(\left(r - \frac{1}{2}\sigma_s^2 - \delta\right) - \lambda^* \mu_J^*\right)(T - t) + \int_t^T b(v)dv + \sigma_s \int_t^T dZ_s^*(v) + \sum_{j=1}^n \ln(Y_j),$$

$$X(T) = \ln S(t) + \left(\left(r - \frac{1}{2}\sigma_s^2 - \delta\right) - \lambda^* \mu_J^*\right)\tau + \int_t^T b(v)dv + \sigma_s \int_t^T dZ_s^*(v) + \sum_{j=1}^n \ln(Y_j),$$

$$S(T) = S(t)\exp\left\{\left(\left(r - \frac{1}{2}\sigma_s^2 - \delta\right) - \lambda^* \mu_J^*\right)\tau + \int_t^T b(v)dv + \sigma_s \int_t^T dZ_s^*(v)\right\} Y(n),$$

$$\frac{S(T)}{S(t)} = \exp\left\{\left(\left(r - \frac{1}{2}\sigma_s^2 - \delta\right) - \lambda^* \mu_J^*\right)\tau + \int_t^T b(v)dv + \sigma_s \int_t^T dZ_s^*(v)\right\} Y(n), \tag{5}$$

Again, as stated in the paper text, where $Y(n) = 1$ if $n = 0$; $Y(n) = \prod_{j=1}^{n} Y_j$ for $n \geq 1$. Also recall, the Y_j are independently and identically distributed and n is Poisson distributed with parameter $\lambda^* \tau$, where $\tau = T - t$.

Appendix 2 Derivation of pricing model (Equation #5)

In this section, we present a complete derivation of pricing model (5) from the paper.

To determine the risk-neutral expected payout for an option, the posited dynamics of the underlying security must be stated. In this case, this standard application is not a straightforward process. The complication centers on the fact that the futures contract itself is a contingent claim, which precludes one from simply stating the price dynamics exogenously. Instead, the stochastic differential for the futures contract must be developed endogenously from the underlying system of state variables that impact the value of the commodity contract. Henceforth, we detail the derivation of pricing model (5) from the text.

Consider the following, if the futures price is a function of the spot price, convenience yield and time, namely $F(SY, b, t)$, then using Ito's lemma, we may express the futures price dynamic as

$$dF = \frac{1}{2}F_{ss}dS^2_{dq=0} + F_s dS_{dq=0} + \frac{1}{2}F_{bb}db^2 + F_b db + F_{sb}dSdb + F_t dt + F(SY, t) - F(S, t). \tag{1}$$

The right-hand side of the aforementioned equation can be rewritten by adding and subtracting $\lambda^* E^*[F(SY, t) - F(S, t)]dt$. This adjustment yields

$$\begin{aligned} dF = {} & \frac{1}{2}F_{ss}dS^2_{dq=0} + F_s dS_{dq=0} + \frac{1}{2}F_{bb}db^2 + F_b db + F_{sb}dSdb + F_t dt + F(SY, t) - F(S, t) \\ & - \lambda^* E^*[F(SY, t) - F(S, t)]dt + \lambda^* E^*[F(SY, t) - F(S, t)]dt, \end{aligned} \tag{2}$$

Substituting for the increments of the spot price and convenience yield and rearranging the aforementioned yields

$$\begin{aligned} dF = {} & \frac{1}{2}F_{ss}\sigma_s^2 S^2 dt + F_s\left[\left(r - \delta + b(t) - \lambda^* \mu_J^*\right)Sdt + \sigma_s S dZ_s^*(t)\right] + \frac{1}{2}F_{bb}\sigma_b^2 dt \\ & + F_b\left[-\kappa b(t)dt + \sigma_b dZ_b^*(t)\right] + F_{sb}\rho_{sb}\sigma_s\sigma_b dt + F_t dt + F(SY, t) - F(S, t) \\ & - \lambda^* E^*\left[F(SY, t) - F(S, t)\right]dt + \lambda^* E^*\left[F(SY, t) - F(S, t)\right]dt \end{aligned} \tag{3}$$

$$\begin{aligned} = {} & \frac{1}{2}F_{ss}\sigma_s^2 S^2 dt + \frac{1}{2}F_{bb}\sigma_b^2 dt + F_s\left(r - \delta + b(t) - \lambda^* \mu_J^*\right)Sdt + F_b[-\kappa b(t)dt] + F_{sb}\rho_{sb}\sigma_s\sigma_b dt \\ & + F_t dt + \lambda^* E^*[F(SY, t) - F(S, t)]dt + F_s\sigma_s S dZ_s^*(t) + F_b\sigma_b dZ_b^*(t) + F(SY, t) \\ & - F(S, t) - \lambda^* E^*[F(SY, t) - F(S, t)]dt, \end{aligned} \tag{4}$$

Which can in turn be restated as equation (5) from the text

$$\begin{aligned} = {} & \left\{\frac{1}{2}F_{ss}\sigma_s^2 S^2 + \frac{1}{2}F_{bb}\sigma_b^2 + F_s\left(r - \delta + b(t) - \lambda^* \mu_J^*\right)S - F_b\kappa b(t) + F_{sb}\rho_{sb}\sigma_s\sigma_b + F_t \right. \\ & \left. + \lambda^* E^*[F(SY, t) - F(S, t)]\right\}dt + F_s\sigma_s S dZ_s^*(t) + F_b\sigma_b dZ_b^*(t) + F(SY, t) - F(S, t) \\ & - \lambda^* E^*[F(SY, t) - F(S, t)]dt. \end{aligned} \tag{5}$$

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